China Flux 2021

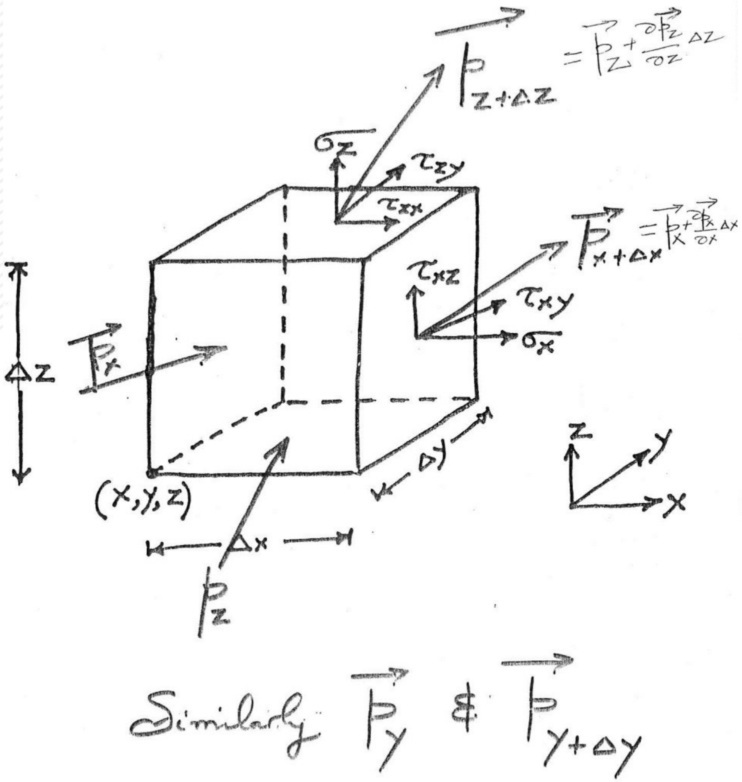
Turbulence Mechanism of Fluid Motion in Atmospheric Boundary Layer and the Similarity Theory Description of its Structure (Part 2)

Outline

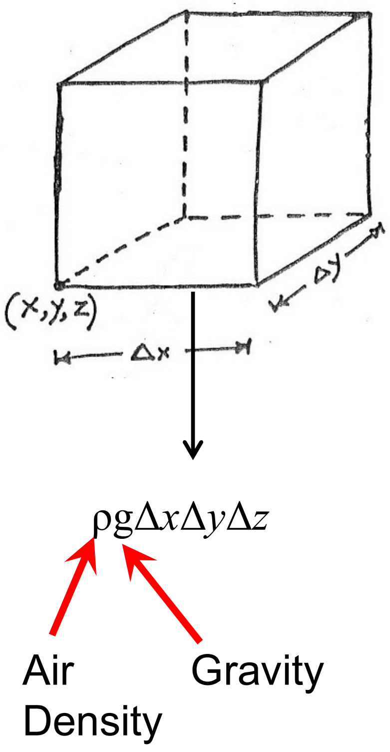
1. Mathematical description of fluid
2. Generation and Dissipation of Turbulence
3. Turbulent transfer of momentum, heat and mass
4. Monin-Obukhov Length and Similarity
5. Turbulent Structure

Physical and Mathematical Description of Fluid Motion

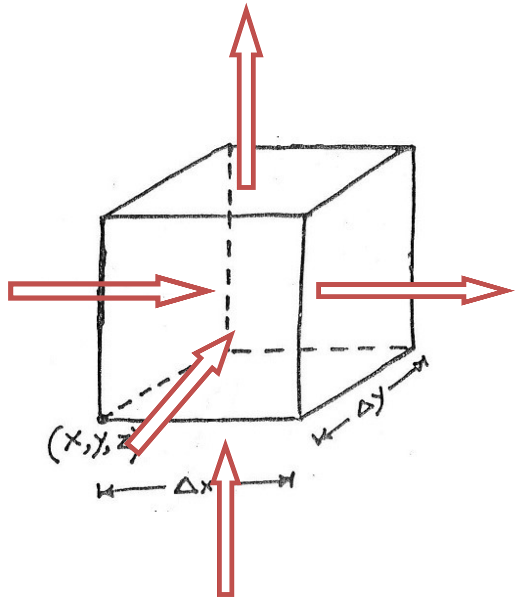
Pressure (*P*) Stress
(τ, σ) Internal



Gravity and density External

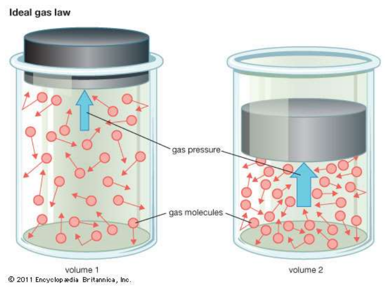


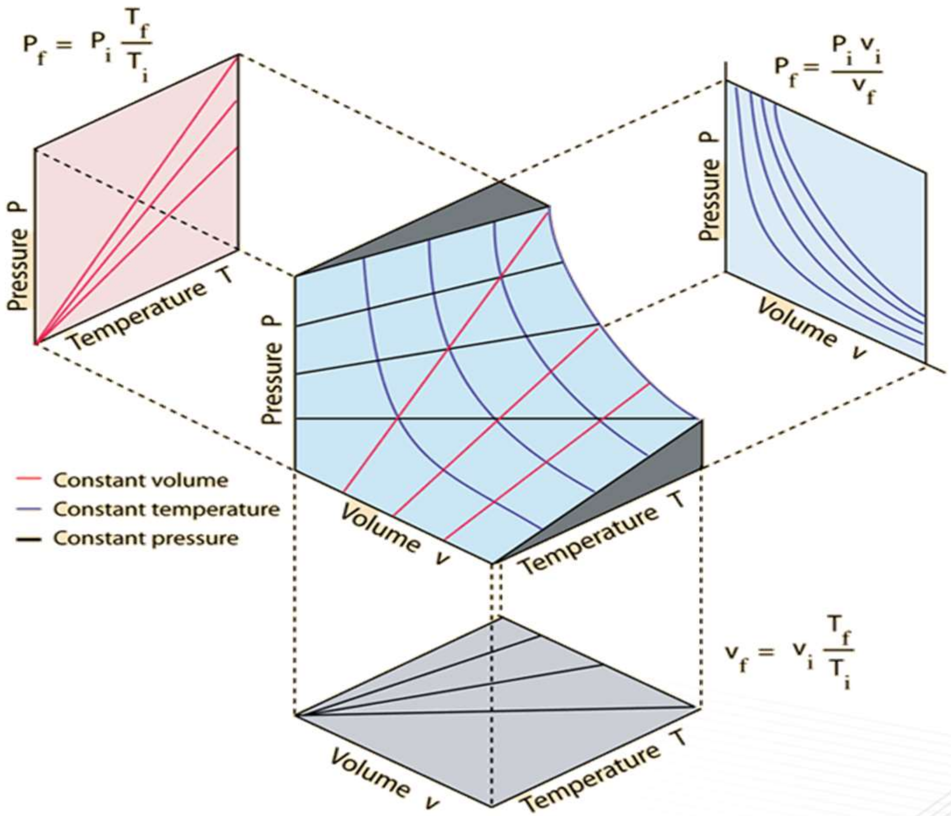
Conservation of Mass



Equation of State

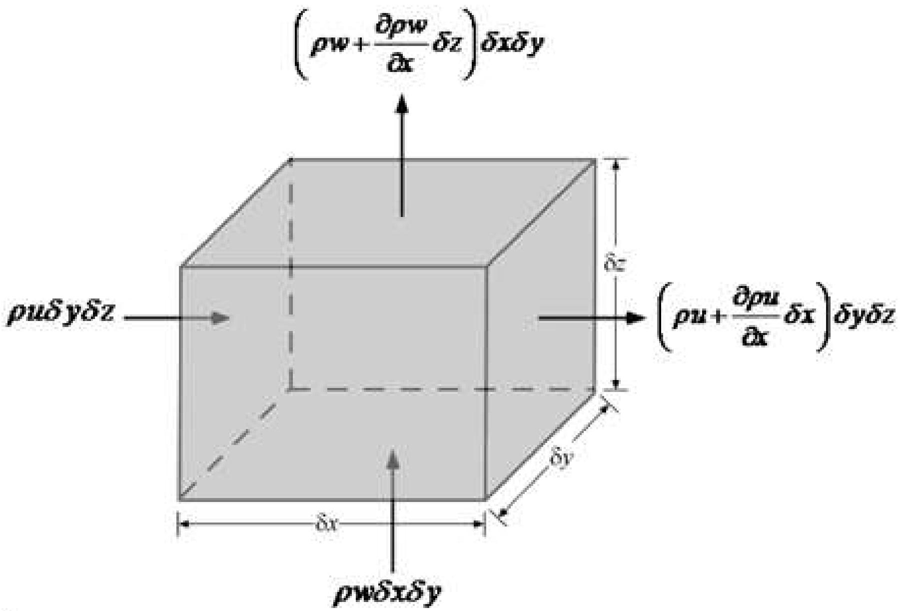
$$P = \rho RT$$

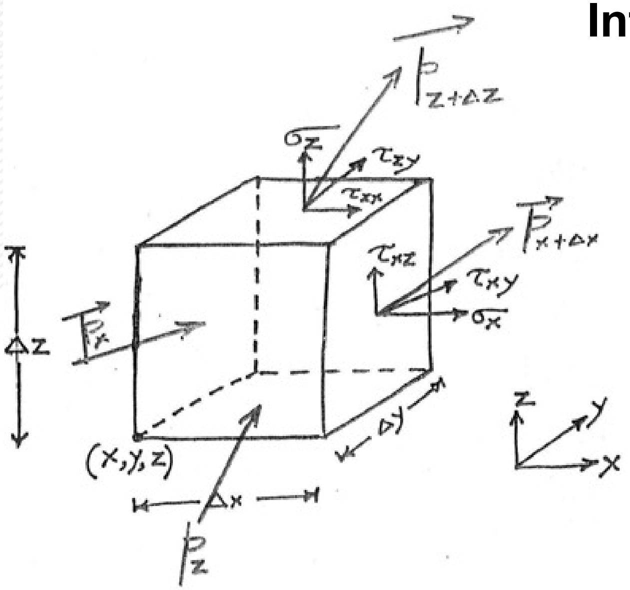




Conservation of Mass Equation

$$\frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$$





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$\rho g \Delta x \Delta y \Delta z$

Air Density

Gravity

Internal Force generated by pressure in the x direction

$$P_x = \left(\vec{P}_{x+\Delta x} - \vec{P}_x\right)\Delta y \Delta z = \frac{\partial \vec{P}}{\partial x}\Delta x \Delta y \Delta z$$

External Force in 3 directions

$$F_x = 0, \quad F_y = 0$$

$$F_z = \rho g \Delta x \Delta y \Delta z$$

Newton's Second Law

$$\rho \frac{du}{dt} = \frac{\partial \vec{P}}{\partial x}$$

$$\rho \frac{dv}{dt} = \frac{\partial \vec{P}}{\partial y}$$

$$\rho \frac{dw}{dt} = \frac{\partial \vec{P}}{\partial z} - \rho g$$

Air Velocity: U direction

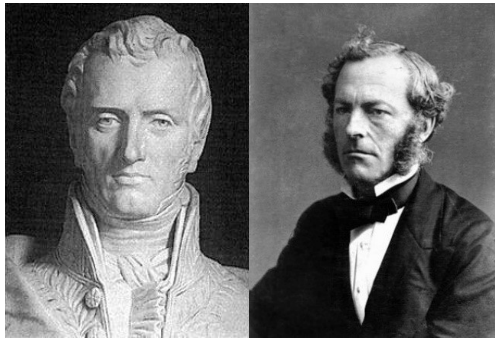
$$u = u(t, x, y, z)$$

$$du = \frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

(Navier-Stokes Equations)



Mass

Acceleration

Forces

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\rho g$$

Tensor notation (X Face)

$$x \rightarrow x_1 \qquad u \rightarrow u_1$$
$$y \rightarrow x_2 \qquad v \rightarrow u_2$$
$$z \rightarrow x_3 \qquad w \rightarrow u_3$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$u_J \frac{\partial u_1}{\partial x_J}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \rho g$$

$$\rho\left(\frac{\partial u_1}{\partial t} + u_1\frac{\partial u_1}{\partial x_1} + u_2\frac{\partial u_1}{\partial x_2} + u_3\frac{\partial u_1}{\partial x_3}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2} + \frac{\partial^2 u_1}{\partial x_3^2}\right)$$

$$\rho\left(\frac{\partial u_2}{\partial t} + u_1\frac{\partial u_2}{\partial x_1} + u_2\frac{\partial u_2}{\partial x_2} + u_3\frac{\partial u_2}{\partial x_3}\right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 u_2}{\partial x_1^2} + \frac{\partial^2 u_2}{\partial x_2^2} + \frac{\partial^2 u_2}{\partial x_3^2}\right)$$

$$\rho\left(\frac{\partial u_3}{\partial t} + u_1\frac{\partial u_3}{\partial x_1} + u_2\frac{\partial u_3}{\partial x_2} + u_3\frac{\partial u_3}{\partial x_3}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 u_3}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_2^2} + \frac{\partial^2 u_3}{\partial x_3^2}\right) - \rho g$$

$$\rho\left(\frac{\partial u_j}{\partial t} + u_J\frac{\partial u_j}{\partial x_J}\right) = -\frac{\partial P}{\partial x_i} + \mu\frac{\partial^2 u_i}{\partial x_J \partial x_J} - \rho g\delta_{3i}$$

$$\begin{cases} \delta_{31} = 0 \\ \delta_{32} = 0 \\ \delta_{33} = 1 \end{cases}$$

Kronecker Delta

Fluid Equations

Equation of State $P = \rho RT$

Continuity $$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u_J}{\partial x_J} = 0$$

Molecular Air Viscosity

Motion $$\frac{\partial u_i}{\partial t} + u_J \frac{\partial u_i}{\partial x_J} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\mu}{\rho}\frac{\partial^2 u_i}{\partial x_J \partial x_J} - g\delta_{3i}$$

Kronecker Delta

(Boussinesque Approximation)

Hypothesis

1. Air viscosity coefficient doesn't change (μ = constant)

2. Air is not Compressible

3. Instantaneous and reference value are relatively small

	Pressure	Temperature	Air Density
Reference	P_0	T_0	ρ_0
Instantaneous	P	T	ρ

$$P_1 = P - P_0 \qquad T_1 = T - T_0 \qquad \rho_1 = \rho - \rho_0$$

$$\left|\frac{P_1}{P_0}\right| \ll 1 \qquad \left|\frac{T_1}{T_0}\right| \ll 1 \qquad \left|\frac{\rho_1}{\rho_0}\right| \ll 1$$

Equation of state

$$P = \rho RT$$

$$\ln P = \ln \rho + \ln R + \ln T$$

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$\frac{dP}{P} = \frac{P - P_0}{P_0 + P_1} = \frac{P_1}{P_0 + P_1} \approx \frac{P_1}{P_0}$$

$$\frac{d\rho}{\rho} \approx \frac{\rho_1}{\rho_0} \qquad \frac{dT}{T} \approx \frac{T_1}{T_0}$$

$$\frac{P_1}{P_0} = \frac{\rho_1}{\rho_0} + \frac{T_1}{T_0}$$

$$\frac{\rho_1}{\rho_0} = -\frac{T_1}{T_0}$$

Approximation of Continuity Equation

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial u_J}{\partial x_J} = 0$$

$$\frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial u_J}{\partial x_J} = 0$$

Approximation of Motion

$$\frac{\partial u_i}{\partial t} + u_J \frac{\partial u_i}{\partial x_J} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\mu}{\rho}\frac{\partial^2 u_i}{\partial x_J \partial x_J} - g\delta_{3i}$$

Net Pressure Equation

$$P(z) = \int_z^{\infty} g\rho\left(h\right)dh$$

$$\frac{\partial P_0}{\partial z} = -\rho_0 g$$

$$\frac{\partial P_0}{\rho_0 \partial x_3} = -g$$

Approximation of the Motion Equation (Mass, Pressure and Force Terms)

$$\frac{\partial \rho}{\partial t} = 0$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial x_i} = -\frac{1}{\rho_0+\rho_1}\frac{\partial (P_0+P_1)}{\partial x_i} = -\frac{1}{\rho_0}\left(1-\frac{\rho_1}{\rho_0}\right)\left(\frac{\partial P_0}{\partial x_i}+\frac{\partial P_1}{\partial x_i}\right)$$

$$\frac{\partial u_J}{\partial x_J} = 0$$

$$\frac{\partial P_0}{\rho_0 \partial x_3} = -g \qquad = -\frac{\partial P_0}{\rho_0 \partial x_i} - \frac{1}{\rho_0}\frac{\partial P_1}{\partial x_i} + \frac{\rho_1}{\rho_0}\frac{\partial P_0}{\rho_0 \partial x_i} + \frac{\rho_1}{\rho_0^2}\frac{\partial P_1}{\partial x_i}$$

$$\frac{\rho_1}{\rho_0} = -\frac{T_1}{T_0} \qquad = g\delta_{3i} - \frac{1}{\rho_0}\frac{\partial P_1}{\partial x_i} - \frac{\rho_1}{\rho_0} g\delta_{3i} - 0$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial x_i} - g\delta_{3i} = -\frac{1}{\rho_0}\frac{\partial P_1}{\partial x_i} + \frac{T_1}{T_0} g\delta_{3i}$$

Approximation Equation of Motion (Velocity, Mass, Viscosity, Force)

$$\frac{\partial u_i}{\partial t} + u_J \frac{\partial u_i}{\partial x_J} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} - g\delta_{3i} + \frac{\mu}{\rho}\frac{\partial^2 u_i}{\partial x_J \partial x_J}$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial x_i} - \quad g\delta_{3i} = -\frac{1}{\rho_0}\frac{\partial P_1}{\partial x_i} + \frac{T_1}{T_0} g\delta_{3i}$$

$$\frac{\partial u_i}{\partial t} + u_J \frac{\partial u_i}{\partial x_J} = -\frac{\partial P_1}{\rho_0 \partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_J \partial x_J} + \frac{T_1}{T_0} g\delta_{3i}$$

Heat Transfer Equation

$$\frac{\partial T}{\partial t} + u_J \frac{\partial T}{\partial x_J} = \frac{k_1}{\rho_0 C_p} \frac{\partial^2 T}{\partial x_J \partial x_J}$$

Molecular motion Equation

$$\frac{\partial s}{\partial t} + u_J \frac{\partial s}{\partial x_J} = D_S \frac{\partial^2 s}{\partial x_J \partial x_J}$$

Equation of Motion Approximation

$$\frac{\partial u_i}{\partial t} + u_J \frac{\partial u_i}{\partial x_J} = -\frac{\partial P_1}{\rho_0 \partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_J \partial x_J} + \frac{T_1}{T_0} g \delta_{3i}$$

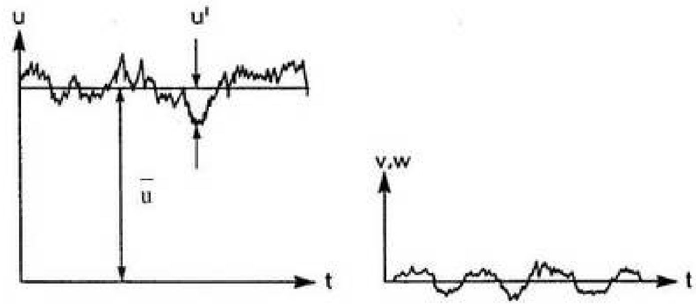
Heat Transfer Equation Approximation

$$\frac{\partial T_1}{\partial t} + u_J \frac{\partial T_1}{\partial x_J} = \frac{k_1}{\rho_0 C_p} \frac{\partial^2 T_1}{\partial x_J \partial x_J}$$

Molecular Motion Approximation

$$\frac{\partial s}{\partial t} + u_J \frac{\partial s}{\partial x_J} = D_S \frac{\partial^2 s}{\partial x_J \partial x_J}$$

Reynold's Averaging



Decomposing Rule $u_i = \bar{u} + u_i'$

$\overline{\bar{u} + u'} = \bar{\bar{u}} + \overline{u'} = \bar{u}$

Averaging Rules

$$\bar{u} = \frac{1}{n}\sum_{n=1}^{n} u_i$$

$$\frac{\overline{\partial u}}{\partial x} = \frac{\partial \bar{u}}{\partial x}$$

$$\overline{u'} = \frac{1}{n}\sum_{i=0}^{n} u_i' = \frac{1}{n}\sum_{i=0}^{n} u_i - \frac{1}{n}\sum_{i=0}^{n} \bar{u} = 0$$

$$\overline{uw} = \overline{(\bar{u} + u')(\bar{w} + w')}$$

$$= \bar{u}\bar{w} + \overline{u'w'}$$

Equation of motion to Turbulence

Boussinesque Approximation equation of motion

$$\frac{\partial u_i}{\partial t} + u_J \frac{\partial u_i}{\partial x_J} = -\frac{\partial p_1}{\rho_0 \partial x_i} + \frac{\mu}{\rho_0} \frac{\partial^2 u_i}{\partial x_J \partial x_J} + \frac{T_1}{T_0} g \delta_{3i}$$

$$\frac{\partial(\bar{u} + \mathrm{u}_i')}{\partial t} + (\bar{u}_J - u_J') \frac{\partial(\bar{u}_i + \mathrm{u}_i')}{\partial x_J} = -\frac{\partial(\bar{p}_1 + p_1')}{\rho_0 \partial x_i} + \frac{\mu}{\rho_0} \frac{\partial^2(\bar{u}_i + \mathrm{u}_i')}{\partial x_J \partial x_J} + \frac{\bar{T}_1 - \theta'}{T_0} g \delta_{3i}$$

↓

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_J \frac{\partial \bar{u}_i}{\partial x_J} = -\frac{\partial \bar{p}_1}{\rho_0 \partial x_i} + \frac{1}{\rho_0} \frac{\partial}{\partial x_J} \left(\mu \frac{\partial \overline{u_i}}{\partial x_J} - \rho_0 \overline{u_i' u_J'} \right) + \frac{\bar{T}_1}{T_0} g \delta_{3i}$$

Momentum Flux

$$\frac{\partial \bar{u}_1}{\partial t} + \bar{u}_J \frac{\partial \bar{u}_1}{\partial x_J} = -\frac{\partial \bar{p}_1}{\rho_0 \partial x_1} + \frac{1}{\rho_0}\frac{\partial}{\partial x_J}\left(\mu \frac{\partial \bar{u}_1}{\partial x_J} - \rho_0 \overline{u_1' u_J'}\right) + \frac{\bar{T}_1}{T_0} g\delta_{31}$$

$$\frac{\partial \rho}{\partial t} = 0$$

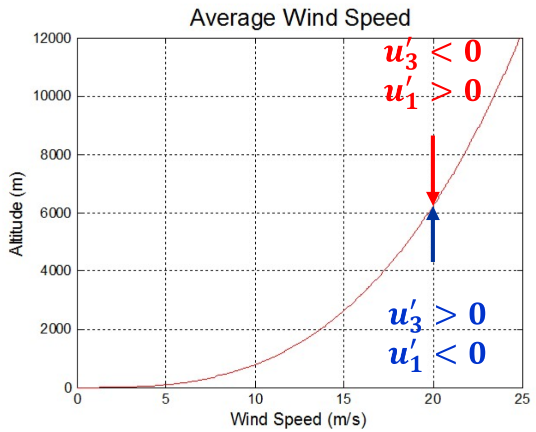
$$\bar{u}_3 \frac{\partial \bar{u}_1}{\partial x_3} = \frac{1}{\rho_0}\frac{\partial}{\partial x_3}\left(\mu \frac{\partial \bar{u}_1}{\partial x_3} - \rho_0 \overline{u_1' u_3'}\right)$$

$$\frac{\partial u_J}{\partial x_J} = 0$$

$$0 = \frac{1}{\rho_0}\frac{\partial}{\partial x_3}\left(\mu \frac{\partial \bar{u}_1}{\partial x_3} - \rho_0 \overline{u_1' u_3'}\right)$$

$$\mu \frac{\partial \bar{u}_1}{\partial x_3} - \rho_0 \overline{u_1' u_3'} = constant$$

$$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$



$-\rho_0 \overline{u_1' u_3'} \approx constant > 0$ (kg m/s)/$(m^2 s)$

Momentum Flux

$$\tau = -\rho_0 \overline{u' w'}$$

$$\frac{\partial \rho}{\partial t} = 0$$

CO2 Flux

$$\frac{\partial \rho_{co_2}}{\partial t} + u_J \frac{\partial \rho_{co_2}}{\partial x_J} = D_{co_2} \frac{\partial^2 \rho_{co_2}}{\partial x_J \partial x_J}$$

$$\frac{\partial u_J}{\partial x_J} = 0$$

$$\frac{\partial(\overline{\rho}_{CO_2} - \rho'_{CO_2})}{\partial t} + (\overline{u}_J + u'_J)\frac{\partial(\overline{\rho}_{CO_2} - \rho'_{CO_2})}{\partial x_J} = D_{co_2}\frac{\partial^2(\overline{\rho}_{CO_2} - \rho'_{CO_2})}{\partial x_J \partial x_J}$$

$$\frac{\partial \overline{\rho}_{CO_2}}{\partial t} + \overline{u}_J \frac{\partial \overline{\rho}_{CO_2}}{\partial x_J} = -\frac{\partial}{\partial x_J}\left(-D_{co_2}\frac{\partial \overline{\rho}_{CO_2}}{\partial x_J} + \overline{\rho'_{co_2} u'_J}\right)$$

$$0 = -\frac{\partial}{\partial x_3}\left(-D_{co_2}\frac{\partial \overline{\rho}_{CO_2}}{\partial x_3} + \overline{\rho'_{co_2} u'_3}\right)$$

$$-D_{co_2}\frac{\partial \overline{\rho}_{CO_2}}{\partial x_3} + \overline{\rho'_{co_2} u'_3} = Constant\ \ \mathrm{mg/(m^2\ s)}$$

CO2 Flux
$$\mathrm{F}_c = \overline{\rho'_{co_2} w'}$$

Momentum Flux

$$\tau = -\rho_0 \overline{u' \, w'}$$

Heat Flux

$$H = \rho_0 C_p \overline{T' \, w'}$$

Latent Heat Flux

$$LE = L\overline{\rho'_{h20} \, w'}$$

Latent Heat Flux

$$LE = L\frac{M_w \rho_d}{M_d}\overline{\chi'_{h20} \, w'}$$

CO2 Flux

$$Fc = \overline{\rho'_{c02} \, w'}$$

CO2 Flux

$$Fc = \frac{\rho_d}{M_d}\overline{\chi'_{co2} \, w'}$$

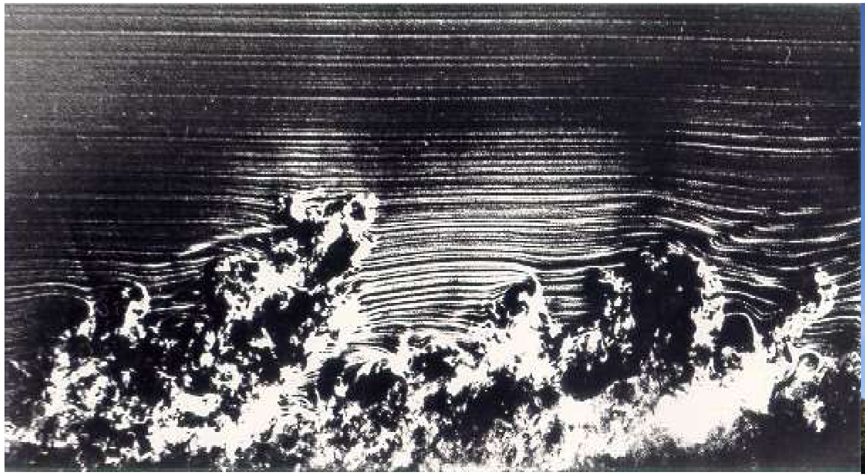
Trace Gas Flux

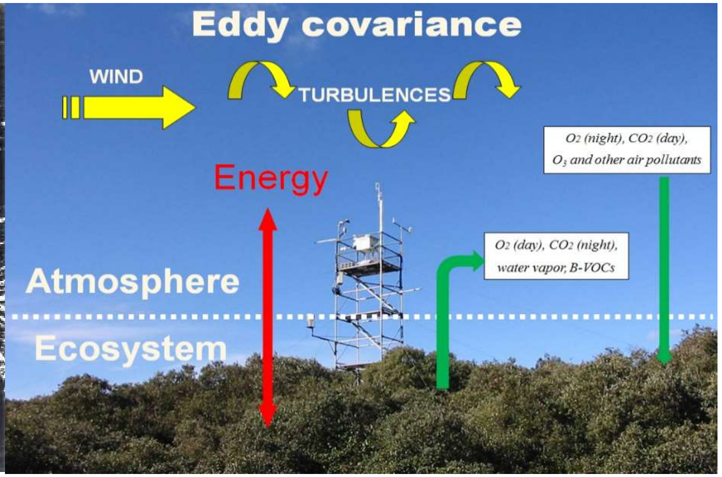
$$F_t = \overline{\rho'_t \, w'}$$

Trace Gas Flux

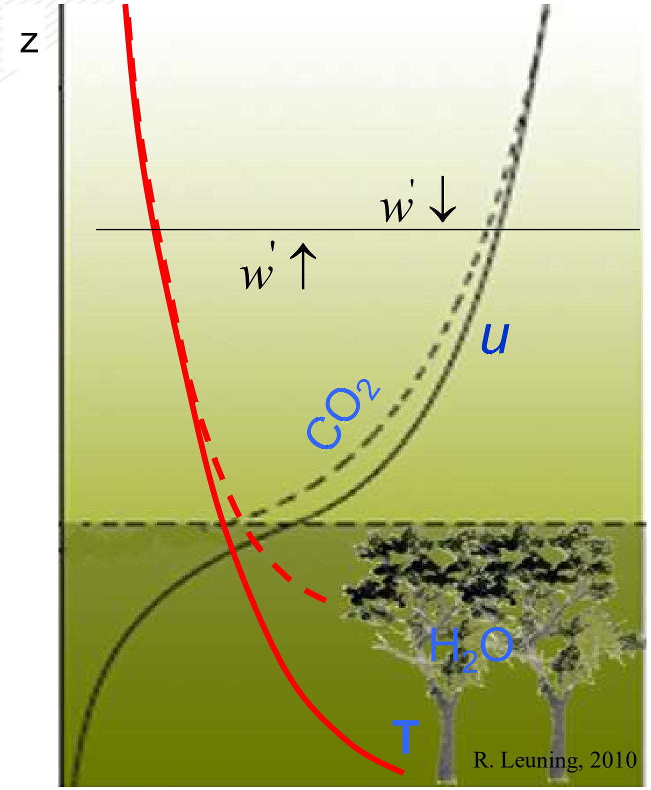
$$F_t = \frac{\rho_d}{M_d}\overline{\chi'_t \, w'}$$

$$F_x = C\,\overline{x'\,w'}$$





Example: Calculate CO_2 Flux



$$F_C = \overline{w'\rho'}_{co2}$$

5Hz

i	ρ'_{co_2} mg m³	w' m s⁻¹	$\rho'_{co_2}w'$ mg m⁻² s⁻¹
1	- 20	0.2	-4
2	10	-0.1	-1
3	-30	0.1	-3
4	20	-0.2	-4
5	20	0	0
$\sum_{i=1}^{5} \rho'_{co_2}w'$			-12
$F_C = \overline{\rho'_{co_2}w'} = \frac{1}{5}\sum_{i=1}^{5} \rho'_{co_2}w'$			-2.4

The Generation and Dissipation of Turbulence

Turbulence Generation and Dissipation (General Expression)

$$e = \frac{\rho}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)$$

$$\frac{de}{dt} = 0$$ **No turbulence generation of dissipation**

$$\frac{de}{dt} > 0$$ **Turbulence dissipation**

$$\frac{de}{dt} < 0$$ **Turbulence dissipation**

Turbulence Generation and Dissipation (Expanded Expression)

$$\frac{De}{Dt} = -\rho\overline{(u'w')}\frac{\partial\bar{u}}{\partial z} + \rho\frac{g}{\overline{T}}\overline{(T'w')} + \frac{\partial\overline{(p'w')}}{\partial z} - \frac{\rho}{2}\frac{\partial\overline{ew'}}{\partial z} - \rho\epsilon$$

- Kinetic Energy Change
- Underlying Surface Friction
- Air mass up and down
- Pressure transport
- Turbulent transport
- Viscosity

Turbulence Flow: Air mass rises and lessoning of viscosity

Turbulence dissipation: Air mass sinks and viscosity increases

Turbulence Transport: Pressure and turbulent transport

Length of Monin Obukhov and Stability of Atmospheric Boundary Layer

$$\frac{De}{Dt} = -\rho\overline{(u'w')}\frac{\partial \bar{u}}{\partial z} + \rho\frac{g}{\bar{\bar{T}}}\overline{(T'w')} + \frac{\partial\overline{(p'w')}}{\partial z} - \frac{\rho}{2}\frac{\partial\overline{ew'}}{\partial z} - \rho\epsilon$$

$$u_* = [(\overline{u'w'})^2\ ]^{\frac{1}{4}}$$

Near-surface Stability

Monin-Obukhov length

$$\frac{z}{L} = -\frac{(g/\overline{T})\overline{T'w'}}{{u_*}^3/kz}$$

$$L = -\frac{{u_*}^3/k}{(g/\overline{T})\overline{(T'w')_0}}$$

Diminsionless

Application of Atmospheric Boundary Layer Stability

$$\frac{z}{L} = -\frac{(g/\overline{T})\overline{T'w'}}{u_*{}^3/kz}$$

According to Monin-Obukhov hypothesis, various atmospheric parameters and statistics such as gradients, variances, and covariance; when normalized the scaling velocity u_* and scaling temperature T_* ; <u>become universal functions of z/L</u> [p15,Kaimal & Finnigan(1994).

$$u_* = \left[-\left(\overline{u'w'}\right)_0\right]^{1/2}$$

$$T_* = \frac{-\left(\overline{w'T'}\right)_0}{u_*}$$

Wind speed and temperature gradients are functions of the stability

p14, Kaimal & Finnigan(1994)

$$\frac{\partial u}{\partial z} = \frac{u_*}{kz}\begin{cases}\left(1+16\left|\frac{z}{L}\right|\right)^{1/4} & -2 \le \frac{Z}{L} \le 0 \\ \left(1+5\frac{z}{L}\right) & 0 \le \frac{z}{L} \le 1\end{cases}$$

$$\frac{\partial \theta}{\partial z} = \frac{T_*}{kz}\begin{cases}\left(1+16\left|\frac{z}{L}\right|\right)^{1/2} & -2 \le \frac{Z}{L} \le 0 \\ \left(1+5\frac{z}{L}\right) & 0 \le \frac{z}{L} \le 1\end{cases}$$

Wind speed and temperature fluctuations are functions of the stability

p14, Kaimal & Finnigan(1994)

$$\sigma_w = u_* \begin{cases} 1.25\left(1+3\left|\frac{z}{L}\right|\right)^{1/3} & -2 \le \frac{Z}{L} \le 0 \\ 1..25\left(1+0.2\frac{z}{L}\right) & 0 \le \frac{z}{L} \le 1 \end{cases}$$

$$\sigma_\theta = T_* \begin{cases} 2\left(1+9.5\left|\frac{z}{L}\right|\right)^{-1/3} & -2 \le \frac{Z}{L} \le 0 \\ .2\left(1+0.5\frac{z}{L}\right)^{-1} & 0 \le \frac{z}{L} \le 1 \end{cases}$$

Turbulent energy dissipation is a function of stability

p14, Kaimal & Finnigan(1994)

$$\varepsilon = \frac{u_*^3}{kz}\begin{cases}\left(1+0.5\left|\frac{z}{L}\right|^{2/3}\right)^{3/2} & -2 \le \frac{Z}{L} \le 0 \\ \left(1+5\frac{z}{L}\right) & 0 \le \frac{z}{L} \le 1\end{cases}$$

Table 1. Velocity scales (friction velocity, u_*, and convective velocity scale, w_*), Obukhov length (L), and planetary boundary layer height (h) characterising the stability regimes of LPDM-B simulations at measurement height z_m and with roughness length z_0. Cases with measurement height within the roughness sublayer were disregarded (see text for details).

Scenario		u_* [m s^{-1}]	w_* [m s^{-1}]	L [m]	h [m]
1	convective	0.2	1.4	−15	2000
2	convective	0.2	1.0	−30	1500
3	convective	0.3	0.5	−650	1200
4	neutral	0.5	0.0	∞	1000
5	stable	0.4	–	1000	800
6	stable	0.4	–	560	500
7	stable	0.3	–	130	250
8	stable	0.3	–	84	200

Receptor heights at $z_m/h = [0.005, 0.01, 0.075, 0.25, 0.50]$
Roughness lengths $z_0 = [0.01, 0.1, 0.3, 1.0, 3.0]$ m

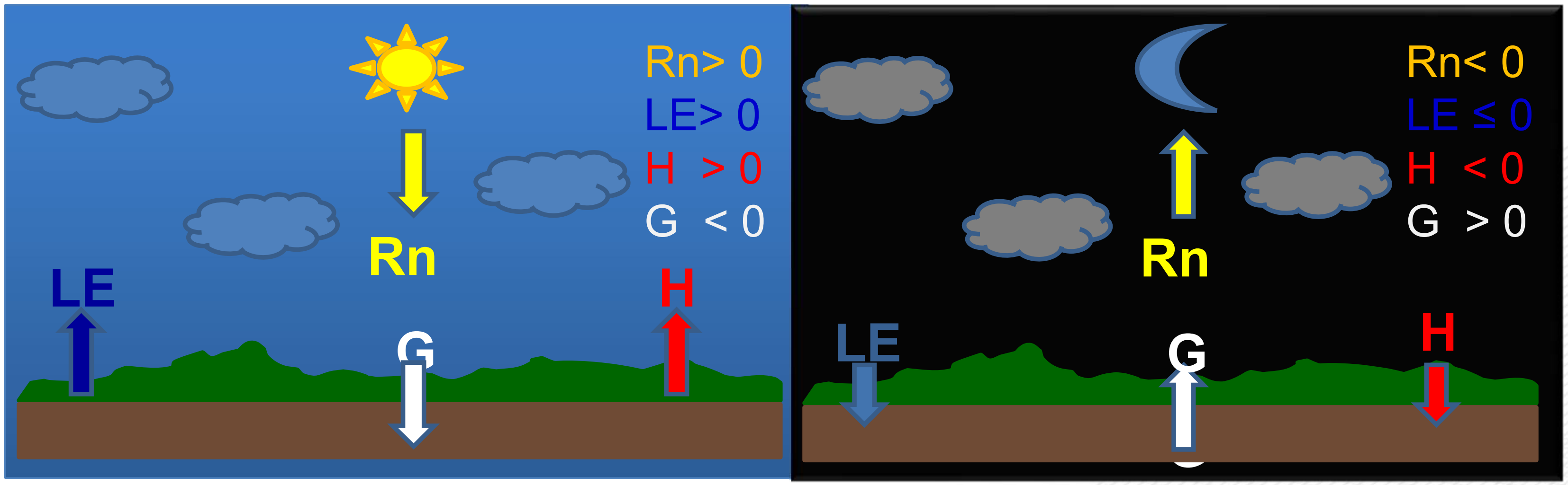
Kljun et al (2015)

Near-surface stability

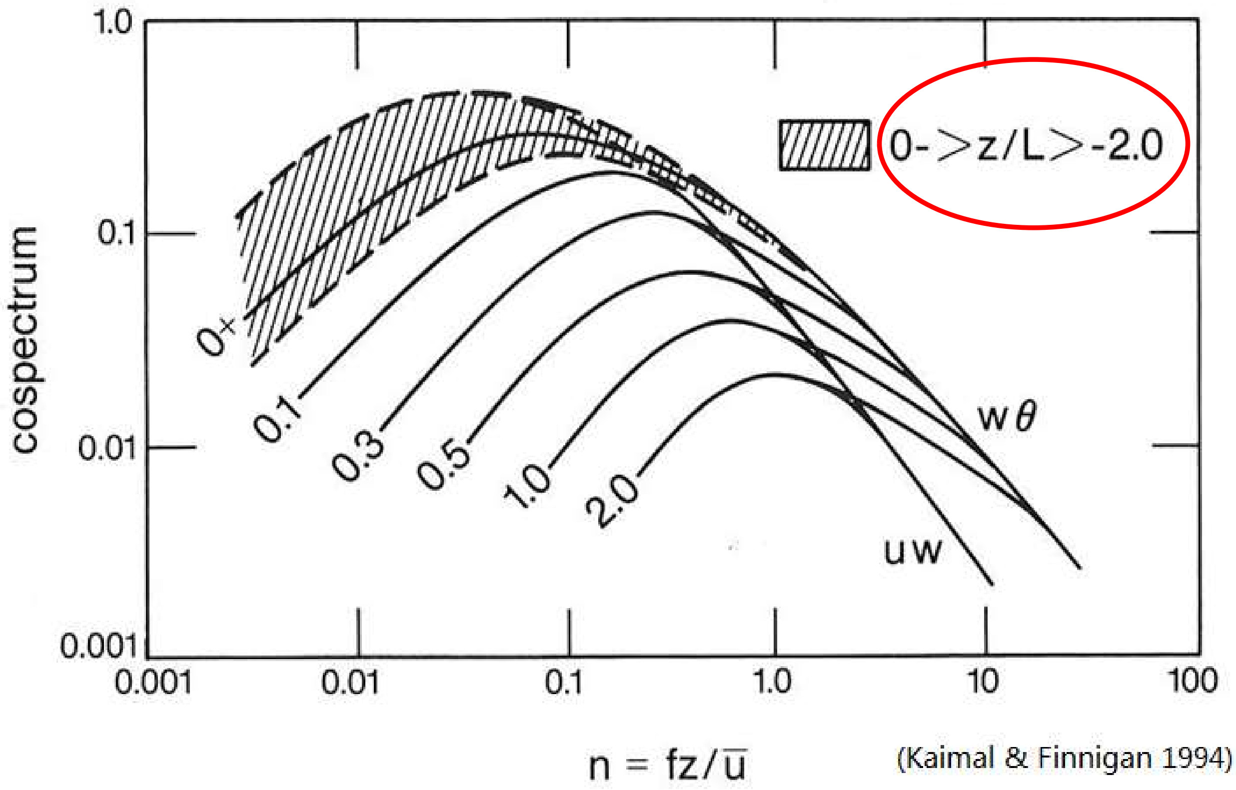
$$\frac{z}{L} = -\frac{\left(g_0 / \overline{T}\right)\overline{w'T'}}{u_*^3 / kz}$$

$\frac{z}{L} < 0$ *unstable*

$\frac{z}{L} > 0$ *stable*



Co-spectrum of vertical and horizontal wind speed under differing stability



(Kaimal & Finnigan 1994)

References

Kaimal, J.C. and J.J. Finnigan. 1994. Atmospheric Boundary Layer Flows: Their Structure and Measurements. Oxford University Press, New York, NY. 289 pp.

Lumley, J.L. and Panofsky. 1964. The structure of Atmospheric Turbulence. John Wiley and Sons, Inc. 239 pp.

Wallace, J.M. and P.V. Hobbs. 2006. Atmospheric Science: An introductory Survey.

Schlichting, H. 1960. Boundary Layer Theory. 4th ed. McGraw-Hill Co. , Inc. 647 pp.

Stull, R.B. 1988. An introduction to Boundary-Layer Meteorology. Kluwer Academy Publishers, Boston. 666 pp.

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